

Guilt and Participation*

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Abstract

How does guilt affect participation in providing public goods? We characterise and analyse completely mixed symmetric equilibria (CMSE) in participation games where players are guilt averse. We find that relative to material preferences, guilt aversion can: facilitate the existence of CMSE; increase or decrease participation; and imply that group size has a non-monotonic effect on participation. Using our equilibrium characterisation we also re-analyse experimental data on participation games and find a low, but positive, guilt sensitivity parameter.

Keywords: Participation, threshold public good, volunteer's dilemma, psychological games, guilt aversion.

JEL codes: C72, H41

1 Introduction

Social dilemmas involve conflicts between private and group incentives. One important class of social dilemma involves the decision to participate in the provision of a discrete public good (Palfrey and Rosenthal 1984, P&R). In this setting, which we refer to as a participation game, each individual decides whether to participate, and when a threshold number of participants is reached, the public good is provided. The decision to participate is characteristic of strategic and economic situations ranging from the volunteer's dilemma to referendum voting to international agreements such as the Kyoto Protocol. Social dilemmas such as public goods games and participation games typically have Nash equilibria involving no cooperation. Yet evidence from both the lab and field suggests that people do contribute to public goods, volunteer, and participate.

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One such motivation for participation may be the desire to avoid disappointing or “letting down” one’s co-players, also known as guilt aversion (Dufwenberg 2002; Charness & Dufwenberg 2006; Battigalli & Dufwenberg 2007). Emotions such as guilt are thought by psychologists to be central to the facilitation of social behavior (Chang & Smith 2015). In addition, guilt aversion may provide a microfoundation for the decision to participate, as the shared expectation of contributing to the public good may constitute a moral expectation or social norm (Charness & Dufwenberg 2006). Because guilt aversion is a belief-dependent motivation, in this paper we employ the toolbox of psychological game theory (Geanakoplos, Pearce & Stacchetti 1989, Battigalli & Dufwenberg 2009) to study how guilt affects the decision to participate.

In general, participation games have many asymmetric Nash equilibria, both in pure and mixed strategies. Many authors have thus argued that completely mixed symmetric equilibria (CMSE) are attractive (easier to learn and coordinate on, e.g. P&R 1984; Dixit & Olsen 2000). We focus on such equilibria and explore three issues: equilibrium existence; comparative statics; and empirical estimates of guilt aversion.

For the general class of participation games with an arbitrary provision threshold and number of players, we show that, as with material preferences, with guilt aversion, there exist at most two CMSE. With a few exceptions that we discuss below, comparative statics results carry through from the material participation game to the participation game with guilt aversion. Our analysis demonstrates that despite the additional complexities resulting from belief-dependent motivations, guilt aversion generates largely intuitive results.

In the volunteer’s dilemma (the participation game with a provision threshold of one), guilt aversion implies a unique CMSE in which the probability of participation is increasing in the guilt sensitivity parameter. The equilibrium participation rate is decreasing in the participation cost and increasing in the public good benefit. In contrast to the material game, for high cost-benefit ratios, the equilibrium probability of participating can be increasing in the number of players in the game. However, for lower cost-benefit ratios, equilibrium participation rates are decreasing in the number of players, as in the material payoff game.

In participation games with a provision threshold greater than one, for high cost-benefit ratios where no CMSE exist in the material game, guilt aversion can imply existence of generically two CMSE. As in the game with material preferences, when there are two equilibria an increase in the cost-benefit ratio decreases (increases) the equilibrium participation rate in the high (low) participation equilibrium; an increase in the number of players reduces equilibrium participation rates; and an increase in the provision threshold increases equilibrium participation rates. An increase in guilt sensitivity decreases participation in the low participation equilibrium and increases it in the high participation equilibrium.

Using our equilibrium characterisation, we re-analyse existing experiments on par-

participation games to estimate the average guilt sensitivity parameter. We contrast our estimates with existing measures in the literature to comment on the portability of guilt aversion across different strategic environments. Although our estimates vary widely, they are, largely, positive and in a range between 0 and 1. The empirical analysis is especially straightforward for the volunteer’s dilemma, for which both the material game and the game with guilt aversion have a single CMSE. In general participation rates in these games are somewhat greater than the material CMSE prediction, consistent with the idea that guilt aversion and the concern for other’s disappointment may motivate participation. We do not suggest that guilt aversion is the sole factor driving participation, but this analysis provides empirical support for the idea that guilt aversion motivates the decision to participate.

Our work adds to a literature studying behavioural preferences in participation games. Existing contributions include Palfrey & Rosenthal (1988) on altruism, Pérez-Martí & Tomás (2004) on warm-glow and regret, and Dufwenberg & Patel (2017) on reciprocity. In addition, a small literature looks at the implications of guilt in linear public good games including Dufwenberg *et al.* (2011) and Dhimi *et al.* (forthcoming).

Rothenhäusler *et al.* (2018) also study guilt in participation games. Their notion of shared guilt is, however, substantially different from ours. They model agents who experience (belief-independent) guilt from supporting an immoral activity, where preferences are private information. By contrast, we apply a psychologically grounded, complete information model of belief-dependent guilt to a standard participation game.

Experimental studies of guilt aversion, including Charness & Dufwenberg (2006), Vanberg (2008), Ellingsen *et al.* (2009), and Khalmetski *et al.* (2015), and Bellemare *et al.* (2017) largely focus on reduced-form analyses of the relationship between second-order beliefs and behaviour in trust or dictator games. A few papers estimate guilt sensitivities directly. Attanasi *et al.* (2016) estimate guilt sensitivities using data from a laboratory trust game. Bellemare *et al.* (2011) report estimated guilt sensitivities from a four-player game. We will discuss how our estimates of guilt sensitivity compare with such studies later.

We proceed as follows. Section 2 presents P&R’s participation game and Section 3 our model of guilt based on Battigalli & Dufwenberg (2007). Section 4 contains our theoretical results on the effect of guilt on participation: implicitly characterising equilibria (4.1), understanding existence (4.2) and comparative statics (4.3). Section 5 applies our analysis to existing experimental data to estimate guilt sensitivities and Section 6 concludes.

2 The participation game

Consider the following participation game as in P&R. Let $N = \{1, \dots, n\}$ denote the set of players. The set of strategies available to each player i is $S_i = \{0, 1\}$. Each player i chooses a binary strategy $s_i \in S_i$, with strategy profiles given by $s = (s_1, \dots, s_n)$. We

refer to $s_i = 1$ as *participate* and $s_i = 0$ as *abstain*. The threshold for provision of the public good is $w \in [1, n)$.¹ Let $\sum_i s_i = m$ refer to the number of participants.

A mixed strategy for player i is a probability distribution $\sigma_i \in \Delta(S_i)$, where $\Delta(X)$ denotes the collection of probability measures over the set X . The profile of strategies of all but player i is given by $\sigma_{-i} = (\sigma_j)_{j \neq i}$, and the complete profile of mixed strategies for all players is given by $\sigma = (\sigma_i)_{i \in N}$. A mixed strategy Nash equilibrium is a strategy profile σ^* such that for every player i , every action in the support of σ_i^* is a best response to σ_{-i}^* . We assume that players do not actually randomize, but that randomized choices may be interpreted as an expression of players' beliefs. We defer a formal presentation of beliefs to the next section, where we make this interpretation explicit.

Payoffs are given by $\pi_i(s)$ as follows:

$$\pi_i(s_i, s_{-i}) = \begin{cases} v & \text{if } s_i = 0 \text{ and } m \geq w \\ 0 & \text{if } s_i = 0 \text{ and } m < w \\ v - c & \text{if } s_i = 1 \text{ and } m \geq w \\ -c & \text{if } s_i = 1 \text{ and } m < w \end{cases} \quad (1)$$

where $0 < c < v$.

The participation game has many pure strategy Nash equilibria where exactly $m = w$ players choose to participate and the rest abstain; if $w > 1$, there is also one where all players abstain. It also has many mixed strategy Nash equilibria where there are three types of players: those who participate with probability 1, those who abstain with probability 1, and those who participate with probability $\sigma_i \in (0, 1)$.

Given the many asymmetric equilibria, we focus on completely mixed symmetric equilibria throughout (CMSE): that is, we consider equilibria where players view the strategy choices of their co-players as independent and identically distributed random variables. To define the CMSE of P&R's game, we assume that every player i believes that each of his $n - 1$ co-players independently chooses to participate with probability p , so that $\sigma_i = p$ for all i , and denote the probability of abstaining by $q = 1 - p$. Under these assumptions, the number of participants (m) has a binomial distribution. Let $\rho(w; n, p)$ be the probability of a player being pivotal for provision of the public good (i.e. the probability that exactly $m = w - 1$ of i 's $n - 1$ co-players choose to participate). Using the binomial probability mass function,

$$\rho(w; n, p) = \binom{n-1}{w-1} p^{w-1} (1-p)^{n-w}. \quad (2)$$

Let $F(k; r, p)$ be the probability that out of r players, k or fewer participate when each participates with probability p . Using the CDF of a random variable that follows a binomial distribution,

$$F(k; r, p) = \sum_{i=0}^k \binom{r}{i} p^i (1-p)^{r-i}. \quad (3)$$

¹Assuming $w < n$ ensures that there are multiple efficient pure strategy Nash equilibria.

Observation 1 (cf. P&R equation 1.3). *For all $n \geq 2$, $n > w \geq 1$ and $v > c > 0$, a CMSE is a probability p^* satisfying*

$$\rho(w; n, p^*) = \frac{c}{v}. \quad (4)$$

Proof: If player i chooses to abstain, then at least w of the remaining $n - 1$ players must choose participate for the good to be provided. Thus i 's expected payoff from abstaining is $(1 - F(w - 1; n - 1, p))v$. If i chooses to participate, then at least $w - 1$ of the remaining $n - 1$ players must choose to participate for the good to be provided. Thus i 's expected payoff of participating is $(1 - F(w - 2; n - 1, p))v - c$. In a CMSE i must be indifferent between his two options. Equating the two expected payoffs gives condition (4). ■

Equation (4) shows that in equilibrium, the probability of being pivotal for provision is equal to the cost-benefit ratio. This makes intuitive and economic sense: in equilibrium, the expected payoff from contributing is equal to the (certain) cost and players are indifferent between the two actions.

An important special case of the model is where $w = 1$, referred to as the *volunteer's dilemma*.

Observation 2. *In the volunteer's dilemma, for all $n \geq 2$ and $v > c > 0$, there exists a unique CMSE. It is described by*

$$p^* = 1 - \left(\frac{c}{v}\right)^{\frac{1}{n-1}}. \quad (5)$$

Proof: Substitute $w = 1$ into (4) and solve for p . ■

In the volunteer's dilemma with material preferences, a unique CMSE exists. To understand equation (5) more intuitively, rewrite it as $q^{n-1} = \frac{c}{v}$. Thus in a CMSE, the cost-benefit ratio of participating is equal to the probability of being pivotal for provision (i.e. the probability that $n - 1$ players do not participate).

Comparative statics are very intuitive. The equilibrium probability of participating is increasing in the benefit of the public good and decreasing in the cost of participation and the number of players (a higher n implies a lower probability of being pivotal for provision, thus less incentive to participate).

For $w \geq 2$, there may be zero, one or two CMSE.

Observation 3 (cf. P&R proposition 2). *For all $n \geq 2$, $n > w \geq 2$ and $v > c > 0$,*

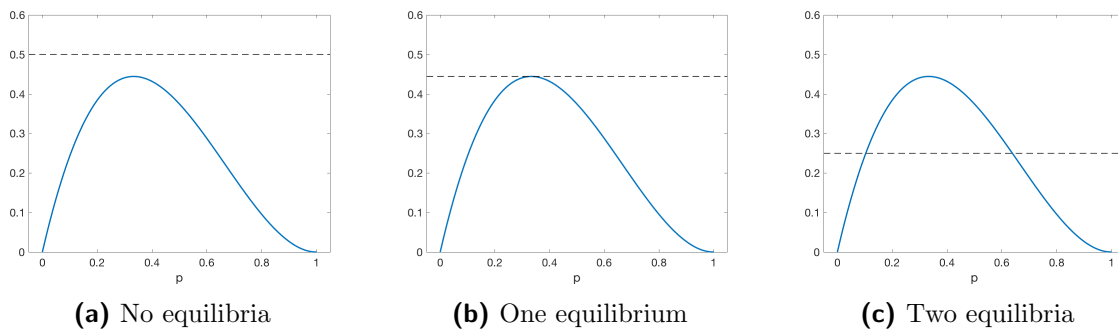
- a. if $\frac{c}{v} > \rho(w; n, \frac{w-1}{n-1})$, then there exists no CMSE;*
- b. if $\frac{c}{v} = \rho(w; n, \frac{w-1}{n-1})$, then there exists one CMSE;*
- c. if $\frac{c}{v} < \rho(w; n, \frac{w-1}{n-1})$, then there exist two CMSE.*

Proof: Recall equilibrium condition (4), $\rho(w; n, p^*) = \frac{c}{v}$. Since $\rho(w; n, p)$ is the probability mass function of a discrete variable following a binomial distribution, defined by (2), $\rho(w; n, 0) = \rho(w; n, 1) = 0$, $\rho(w; n, p)$ is strictly increasing for all $p \in [0, \frac{w-1}{n-1})$ and strictly decreasing for all $p \in (\frac{w-1}{n-1}, 1]$. As $\frac{c}{v} \in (0, 1)$, simply compare $\rho(w; n, \frac{w-1}{n-1})$ with $\frac{c}{v}$ to determine for how many values of p condition (4) holds. ■

For some intuition, reason as follows. If co-players participate with probability one, i should abstain. If they abstain with probability one, i should also abstain. Whether there will exist some participation probability in the middle such that i is indifferent between his two actions depends on c relative to v . If c is too large then although the probability of being pivotal does increase as the probability that co-players participate increases, it does not increase enough, thus no equilibrium exists.

The figure below illustrates the three possible cases.

Figure 1: Equilibria of the participation game with $n = 4$ and threshold $w = 2$.



Note: For $n = 4$ and $w = 2$, the solid line in each panel plots the the probability of being pivotal, $\rho(w; n, p)$, over p . A CMSE is where $\rho(w; n, p) = \frac{c}{v}$. The dashed line in each panel illustrates a different value of $\frac{c}{v}$.

In panel (a) the cost-benefit ratio is so high that no CMSE exist, in panel (b) it is lower so one CMSE exists, and in panel (c) it is lower still so two CMSE exist. Overall, non-existence of CMSE is caused by players not having enough incentive to participate.

Finally, note the comparative statics. Where two CMSE exist, an increase in $\frac{c}{v}$ increases participation in the CMSE with lower participation and decreases it in the CMSE with higher participation. An increase in the number of players, reduces participation in both CMSEs. An increase in w increases participation in both CMSEs.

3 Guilt aversion and psychological Nash equilibrium

Guilt aversion captures the psychological disutility from disappointing one's co-players. In this section we review the notions of guilt aversion (Battigalli & Dufwenberg 2007) and

psychological Nash equilibrium (Geanakoplos *et al.* 1989) as they apply in our setting.

3.1 Guilt aversion

To model guilt aversion, we need to specify players' beliefs. Each player i has a first-order belief $\alpha_i \in \Delta(S_{-i})$ about the strategies of the other players. Let $\alpha_{-i} = (\alpha_j)_{j \neq i}$ denote the first-order beliefs of all players save player i , then $\alpha = (\alpha_i, \alpha_{-i})$ is a profile of first-order beliefs.

Player i also has second-order beliefs β_i about the first-order beliefs of each co-player. Most generally, player i 's second-order beliefs might allow for correlation between the actions of his co-players and their beliefs. Here, we assume that higher-order beliefs are degenerate point beliefs, so that β_i is identified with an array of first-order beliefs: $\beta_i = (\beta_{ij})_{j \neq i}$, where $\beta_{ij} \in \Delta(S_{-j})$. Then $\beta = (\beta_i)_{i \in N}$ is a profile of second-order beliefs.²

Before the game is played, player j can calculate his expected payoff

$$\bar{\pi}_j^0(s_j) = \sum_{s_{-j} \in S_{-j}} \alpha_j(s_{-j}) \pi_j(s_j, s_{-j}),$$

given his strategy s_j and his first-order beliefs α_j about the strategies s_{-j} of the other players. The expression

$$D_j((s_j, s_{-j}), \alpha_j) = \max\{0, \bar{\pi}_j^0(s_j) - \pi_j(s)\} \quad (6)$$

measures how much player j is disappointed or "let down" at the end of the game. After the game is played, if i knew j 's beliefs α_j , he could calculate how much of D_j is due to his own behaviour:

$$G_{ij}(s, \alpha_j) = D_j(s, \alpha_j) - \min_{s'_i} D_j((s'_i, s_{-i}), \alpha_j). \quad (7)$$

A guilt averse player i then chooses s_i to maximize the expected value of

$$u_i((s_i, s_{-i}), \alpha_{-i}) = \pi_i(s) - \sum_{j \neq i} \theta_{ij} G_{ij}(s, \alpha_j), \quad (8)$$

with respect to player i 's second-order beliefs, where $\theta_{ij} \geq 0$ is a parameter capturing player i 's guilt from disappointing player j .³

²Rabin (1993), Dufwenberg & Kirchsteiger (2004) and Battigalli & Dufwenberg (2007) impose a similar condition.

³As noted by Battigalli & Dufwenberg (2007, 2009), a simpler formulation where players dislike other's disappointment: $u_i((s_i, s_{-i}), \alpha_{-i}) = \pi_i(s) - \sum_{j \neq i} \theta_{ij} D_j(s, \alpha_j)$ results in the same best response correspondence. We focus on the definition of guilt in Equations 7 and 8, though we view both guilt and concern for other's disappointment as plausible motivations in participation games.

3.2 Psychological Nash equilibria with guilt aversion

A psychological Nash equilibrium (Geanakoplos, Pearce & Stacchetti 1989) with guilt aversion is a profile of behavior and beliefs such that strategies are best responses to beliefs, and beliefs are correct.

Definition 1. *A psychological Nash equilibrium with guilt aversion is a tuple (σ, α, β) such that*

1. *For each player i and beliefs α_i , for all s_i in the support of σ_i ,*

$$s_i \in \arg \max E_{\alpha_i, \beta_i} [u_i((s_i, s_{-i}), \alpha_{-i})] \quad (9)$$

2. *For all $i \in N$, and for all $j \neq i$, $\alpha_i = \sigma_{-i} = \beta_{ji}$*

As noted above, our focus is on completely mixed symmetric equilibria with guilt aversion.⁴ Therefore, we consider a game where each player i 's preferences are captured by the following utility function:

$$u_i((s_i, s_{-i}), \alpha_{-i}) = \pi_i(s) - \theta G(s, \alpha_{-i}), \quad (10)$$

where $\theta_{ij} = \theta$ and $G(s, \alpha_{-i}) = \sum_{j \neq i} G_{ij}(s, \alpha_j)$.

Definition 2. *A completely mixed symmetric psychological Nash equilibrium with guilt aversion is a psychological Nash equilibrium with guilt aversion (σ, α, β) such that each player believes that all co-players independently choose their strategies, and that for all i, j , and $k \in N$ such that $i \neq j$ and $j \neq k$, $\sigma_i = \alpha_{ji} = \beta_{kji} = p \in (0, 1)$, where α_{ji} denotes the entry corresponding to player i in α_j and β_{kji} is the entry corresponding to player i in β_{kj} .*

Definition 2 says that a CMSE of the participation game with guilt aversion is a psychological Nash equilibrium where each player believes that each other player's participation decision is an independent Bernoulli random variable with probability p . Notice that this definition also includes CMSE of the material participation game, which we obtain by setting $\theta = 0$.

4 Guilt and participation

In this section we characterise and provide comparative statics results for completely mixed, symmetric psychological Nash equilibria of the participation game with guilt aversion. We give special attention to the volunteer's dilemma, for which we provide an analytic solution.

⁴Observation 2 of Battigalli & Dufwenberg (2007) establishes that all of the pure strategy equilibria of the material payoff game are also equilibria of the psychological games with (simple) guilt aversion.

4.1 CMSE with guilt

Our first result allowing for guilt aversion states the condition characterising a CMSE of the participation game.

Proposition 1. *For all $n \geq 2$, $n > w \geq 1$, $v > c > 0$ and $\theta \geq 0$, a CMSE is a p^* satisfying*

$$\rho(w; n, p^*)(1 + \theta\tilde{G}) = \frac{c}{v}, \quad (11)$$

where $\tilde{G} = G/v = (n - 1)(1 - F(w - 1; n - 1, p^*)) + (w - 1)\rho(w; n, p^*)$.

Proof: See appendix.

The left hand side of equation (11) is the expected *marginal benefit of participating* normalised by v . If player i is not pivotal, his marginal benefit of participating is zero. If i is pivotal (which occurs with probability $\rho(w; n, p^*)$) and he participates, his normalised material marginal benefit is 1 and his normalised guilt-alleviation marginal benefit is $\theta\tilde{G}$. The right hand side of equation (11) is the *normalised marginal cost of participating*.

Notice how the equilibrium condition compares to that with material preferences, condition (4). If $\theta = 0$ the two conditions are identical. Each player must be indifferent between participating and not, so the equilibrium probability equates the probability of being pivotal with the material cost-benefit ratio. If $\theta > 0$, conditions (4) and (11) are identical other than $\theta\tilde{G} > 0$ being included in the left-hand side of condition (11).

To understand \tilde{G} , consider when i would feel guilty. The only way that j can feel disappointed is if j expected the good to be provided and it is not. Player i will not feel guilty for j 's disappointment if i participated as there is nothing more i could do to ensure provision. If i abstained, then he only feels guilty for j 's disappointment if i is pivotal for provision (i.e. $w - 1$ other players participate), otherwise i 's choice could not affect provision. Guilt aversion thus endogenously increases the value of participation as by participating i can avoid the possibility of feeling guilty for j 's disappointment.

Notice how \tilde{G} reflects the asymmetry in the guilt that i would feel towards participating and abstaining co-players. If j abstained then he expected that the good would be provided with probability $(1 - F(w - 1; n - 1, p^*))$; if j participated then he expected provision with probability $(1 - F(w - 1; n - 1, p^*)) + \rho(w; n, p^*)$. Thus i would feel more guilty towards each of the $w - 1$ participants than each of the $n - w$ abstainers.

It is difficult to see the implications of guilt for the nature of equilibria in terms of existence, multiplicity and comparative statics from condition (11). We now examine these issues more closely.

4.2 Existence of CMSE with guilt

With material preferences we noted that there exists a unique CMSE in the volunteer's dilemma. The same is true when players are guilt averse.

Proposition 2. *In the volunteer's dilemma, for all $n \geq 2$, $v > c > 0$ and $\theta \geq 0$ there exists a unique CMSE. This CMSE is characterised by*

$$p^*(c, n, v, \theta) = 1 - \left(\frac{2c}{v(1 + \theta(n-1)) + \sqrt{v(v + \theta(n-1)) [v(n-1)\theta - 4(c - \frac{v}{2})]}} \right)^{\frac{1}{n-1}}. \quad (12)$$

Proof: See appendix.

Thus within the class of CMSE, guilt does not create an equilibrium multiplicity problem in the volunteer's dilemma.

The effect of guilt in participation games with a higher threshold is more interesting. Observation 3 stated that with material preferences and $w \geq 2$ there may be zero, one or two CMSE. Allowing for guilt, we have the following result.

Proposition 3. *For all $n \geq 2$, $n > w \geq 2$ and $v > c > 0$, there exists $\theta^* > 0$ such that*

- a. if $\frac{c}{v} \geq \rho(w; n, \frac{w-1}{n-1})$, then for all $\theta > 0$ there exist two CMSE;*
- b. if $\frac{c}{v} < \rho(w; n, \frac{w-1}{n-1})$, then*
 - i. for $\theta > \theta^*$ there exist two CMSE;*
 - ii. for $\theta = \theta^*$ there exists one CMSE;*
 - iii. for $\theta < \theta^*$ there exist no CMSE.*

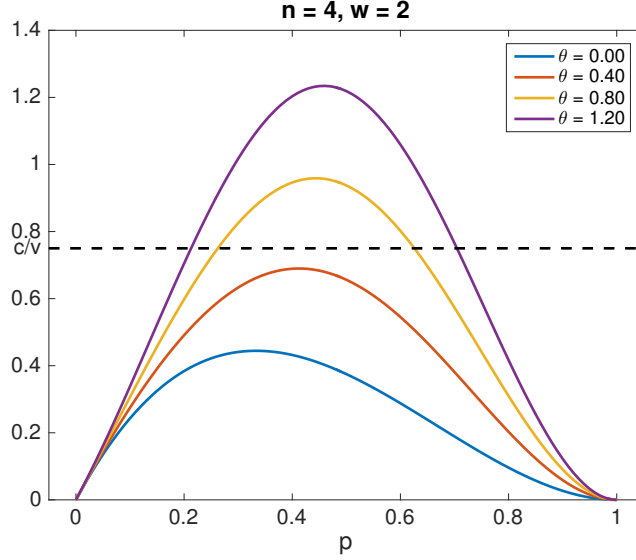
Proof: See appendix.

Contrast how this result differs from Observation 3. There are three cases. If there are two CMSE in the game with material preferences, then there remain two CMSE in the game with guilt. If there is one CMSE in the game with material preferences then there are two CMSE in the game with guilt. If there are no CMSE in the game with material preferences, then if players are only slightly guilt averse then there are no CMSE. However if they are sufficiently guilt averse then there are generically two CMSE. Thus guilt aversion can help the existence of CMSE.

The intuition as to why guilt facilitates existence of CMSE is straightforward. As discussed after Observation 3, non-existence of CMSE with material preferences was caused by too little incentive to participate. Guilt aversion endogenously increases incentives to participate as not doing so can disappoint co-players if the good is not provided and the player is pivotal. If players are sufficiently sensitive to guilt ($\theta \geq \theta^*$), then these incentives to participate are sufficiently high and the existence of a CMSE follows.

The figure below illustrates the point graphically.

Figure 2: The effect of guilt on participation



Note: For $n = 4$ and $w = 2$, this figure plots the normalised marginal benefit of participating over p for various values of θ . A CMSE is a p such that the function equals $\frac{c}{v}$ (an illustrative value of $\frac{c}{v} = \frac{3}{4}$ is depicted).

For the game with $n = 4$ and $w = 2$, the figure plots the normalised expected marginal benefit of participation (LHS of equilibrium condition (11)) over p for various values of θ . A CMSE is a p where this intersects with the normalised marginal cost of participation ($\frac{c}{v}$, RHS of equilibrium condition (11)). For the marginal cost depicted, $\frac{c}{v} = \frac{3}{4}$, and material preferences, i.e. $\theta = 0$, or very low guilt sensitivity, e.g. $\theta = 0.4$, there exist no CMSE. However, when players are more sensitive to guilt, e.g. $\theta = 0.8$, there are two CMSE.

In the introduction we mentioned how coordination may be difficult in participation games with material preferences, due to the large number of pure and mixed strategy Nash equilibria. That guilt aversion can imply only two CMSE when there were zero with material preferences may suggest that guilt averse players are better able to coordinate and provide discrete public goods.

4.3 Comparative statics

Next we consider comparative statics in the volunteer's dilemma with guilt.

Proposition 4. *In the volunteer's dilemma, for all $n \geq 2$, $v > c > 0$ and $\theta \geq 0$,*

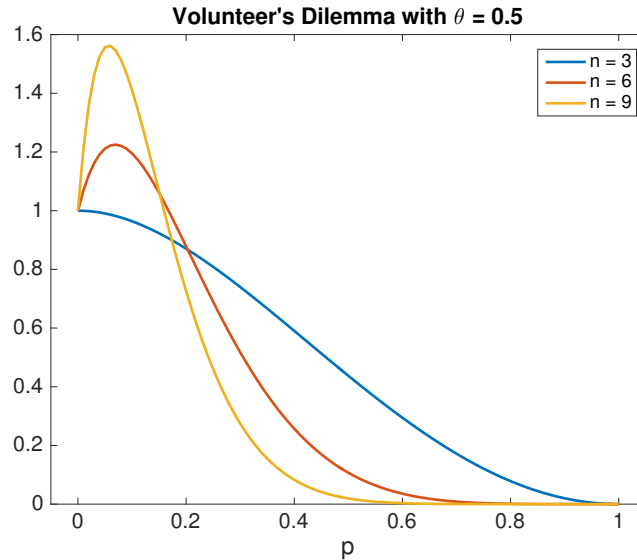
- a. p^* is strictly decreasing in $\frac{c}{v}$,
- b. p^* is strictly decreasing in n , for all $c \leq \bar{c}$,
- c. p^* is strictly increasing in θ .

Proof: See appendix.

The cost-benefit ratio of the public good affects equilibrium participation similarly to how it does with material preferences; it is decreasing in the cost and increasing in the benefit (Proposition 4a). As discussed in Section 4.2, guilt aversion increases incentives to participate so as to avoid disappointing co-players. This is reflected in equilibrium participation increasing in the guilt sensitivity (Proposition 4c).

Recall that with material preferences, CMSE participation in the volunteer's dilemma was strictly decreasing in the number of players (see Observation 2). With guilt, the same is true when the cost is low (Proposition 4b). However, when the cost is high, the relationship is non-monotonic and CMSE participation can be increasing in the number of players. The figure below illustrates.

Figure 3: The effect of the number of players on participation in the volunteer's dilemma



Note: For $w = 1$ and $\theta = 0.5$, this figure plots the normalised marginal benefit of participating over p for various values of n . A CMSE is a p such that the function equals $\frac{c}{v}$.

For $\theta = 0.5$, the figure plots the normalised expected marginal benefit of participation (LHS of equilibrium condition (11)) in the volunteer's dilemma over p for different values of n . A CMSE is a value of p such that the plotted function equals the normalised marginal cost of participation, $\frac{c}{v}$. For the example in the figure, if $\frac{c}{v} < 0.8$ then indeed, CMSE participation is decreasing in the number of players. However, for higher values of $\frac{c}{v}$, CMSE participation can be increasing in the number of players.

To understand the intuition behind why guilt aversion gives rise to the non-monotonic relationship, consider the following two opposing effects. On the one hand, when there are more players, there is a higher probability that someone else will participate, therefore a lower incentive to participate. On the other hand, when there are more players, there are more people that will be disappointed if the good is not provided, thus increasing the guilt that players experience and providing more incentive to participate. When the cost is low, then the first effect always dominates as the material incentive to participate is very high. When the cost is high then which of the two effects is larger depends on the number of players. When there are few players, the first effect dominates. When there are many players, the second effect dominates. Hence the non-monotonic effect of additional players on equilibrium participation in a volunteer's dilemma with a high cost.

Our next result considers comparative statics in the game with a threshold of $w \geq 2$. In order to make a precise statement, we restrict attention to parameters where there exist two CMSE (see Proposition 3 for when this is the case).

Proposition 5. *For all $n \geq 2$, $n > w \geq 2$, $v > c > 0$ and $\theta \geq 0$ where there exist two CMSE, let $p_H^*(c, n, v, w, \theta)$ and $p_L^*(c, n, v, w, \theta)$ denote the CMSE participation probabilities where $p_L^* < p_H^*$, then*

- a. p_L^* is strictly increasing in $\frac{c}{v}$ and p_H^* is strictly decreasing in $\frac{c}{v}$,
- b. p_L^* and p_H^* are strictly decreasing in n ,
- c. p_L^* and p_H^* are strictly increasing in w ,
- d. p_L^* is strictly decreasing in θ and p_H^* is strictly increasing in θ ,

Proof: See appendix.

When there are two CMSE, factors that unambiguously increase the value of participation (i.e. a decrease in the material cost-benefit ratio or an increase in guilt sensitivity) decrease the participation probability for the low participation equilibrium and increase that of the high participation equilibrium. For an illustration of how the equilibria change, see Figure 2.

As in the volunteer’s dilemma, an increase in the number of players has opposing effects. On the one hand, i has more co-players whose disappointment he could feel guilt for, thus increasing his incentive to participate. On the other hand, more co-players means a higher probability of provision, thus a material incentive to decrease participation. The higher probability of provision also implies a lower chance of i feeling guilty, further decreasing his incentive to participate. The latter two effects dominate.

If the provision threshold increases, the probability of provision decreases. Players thus have a material incentive to increase participation in order to compensate for this. Guilt provides additional incentives to increase participation. As discussed after Proposition 1, i only feels guilty if $w - 1$ players participate and feels more guilty towards participants than abstainers. An increase in w increases the number of participants towards whom i feels guilty, thus providing larger guilt-alleviation incentives for i to increase his participation.

5 Estimating guilt sensitivity from experiments

Section 4 illustrated how players’ sensitivity to guilt, θ , can play a critical role in participation games (e.g. determining whether a CMSE exists or not). It is useful to look at data to reflect on the empirical relevance of our results.

We use our equilibrium characterisation and data from existing experimental studies on participation games to impute the value of θ . If these values are drastically different from existing estimates of θ , it calls into question the portability of such estimates for different games and/or the empirical relevance of guilt aversion and CMSE in participation games.

Table 1 presents the value of θ imputed for different studies. The table shows the minimum value of θ required to generate the observed participation rate. Thus, the results in the table give a lower bound on the guilt sensitivity parameter, under the assumption that participants in the experiment are guilt averse with identical sensitivity to guilt.

Table 1: Imputed guilt sensitivities

Study	Treatment	n	w	v	c	Subj.	Observed p	Nash	θ
Dawes et al 1986	Standard	7	3	10	5	70	0.51	none	0.267
	dilemma	7	5	10	5	70	0.64	none	0.131
Rapoport & Eshed-Levy 1989	F&G	5	3	5	2	60	0.365	none	0.152
Erev & Rapoport 1990	Simultaneous	5	3	6	3	35	0.429	0.381 & 0.620	0.205
Diekmann 1993	A	2	1	100	50	33	0.61	0.5	0.462
	H	5	1	100	50	25	0.28	0.159	0.294
Offerman <i>et al.</i> 1996	Low 7	7	3	180	60	63	0.198	none	0.254
	High 7	7	3	245	60	63	0.41	none	-0.054
	High 5 ⁵	5	3	245	60	60	0.504	0.281 & 0.719	-0.153
Cadsby & Maynes 1999	YB1 & GB1	10	5	11	10	20	0.136	none	34.5
	YB2 & GB2	10	5	20	10	20	0.243	none	1.53
	YB3 & GB3	10	5	30	10	20	0.184	none	3.148
	YB4 & GB4	10	5	40	10	20	0.36	0.398 & 0.491	0.026
	YB5	10	5	85	10	10	0.61	0.251 & 0.651	-0.033
Healy & Pate 2009	A, all cost 20	2	1	0.8	0.2	18	0.71	0.75	-0.194
	A, all cost 60	2	1	0.8	0.6	18	0.259	0.25	0.047
	B, all cost 20	6	1	0.8	0.2	18	0.396	0.242	0.459
	B, all cost 60	6	1	0.8	0.6	18	0.137	0.056	0.217
Feldhaus & Stauff 2016	Baseline	3	1	12	6	60	0.288	0.293	-0.014
Goeree <i>et al.</i> 2017	$n = 2$	2	1	0.8	0.2	34	0.52	0.75	-0.921
	$n = 3$	3	1	0.8	0.2	36	0.4	0.5	-0.239
	$n = 6$	6	1	0.8	0.2	48	0.28	0.242	0.072
	$n = 9$	9	1	0.8	0.2	36	0.19	0.159	0.054
	$n = 12$	12	1	0.8	0.2	48	0.61	0.118	0.155

Notes: This table summarises the parameters and participation rates in experiments that run the P&R game. Subj. refers to the number of subjects in the study. Observed p refers to the share of players that chose to participate. *Nash* refers to the equilibrium probability of participation in the CMSE (where it exists) with material preferences given the game parameters. A value of “none” in this column implies no CMSE exists. θ calculates the minimum value of θ needed to generate the observed participation rate assuming subjects play CMSE and given the game parameters.

3. This treatment also appears in Sonnemans *et al.* (1998) as treatment “Public good” and in Offerman *et al.* (2001) as treatment “PGP”.

The mean estimate of θ is 1.68 and the median is 0.142. The subject-weighted mean estimate of θ (a weighted mean estimate of theta where the weight a particular study receives is proportional to the number of subjects it has) is 0.925. Note that there is an obvious outlier (Cadsby & Maynes 1999, YB1 & GB1). Excluding the outlier, the mean is 0.255, the median is 0.131 and the subject-weighted mean is 0.165.

While one cannot over-infer from these back-of-the-envelope calculations for many reasons, a low but positive estimate of guilt sensitivity is consistent with previous work estimating the parameter. Using a four-player game, Bellemare *et al.* (2011) report θ estimates in the range of 0.4 to 0.8. Bellemare *et al.* (2018) use mini-dictator games and find θ in the range of 0.4 to 1.0 when estimates are “stake dependent” and a mean θ of 0.145 for their “stake independent” estimate.⁶ Although not conclusive, our analysis suggests some portability of Battigalli & Dufwenberg’s guilt aversion model to participation games.

6 Conclusion

Participation is necessary for the provision of public goods the world over. Threshold public goods come with an additional challenge, the coordination problem that results from equilibrium multiplicity. We studied the effect of an important motivation that may explain participation in such situations, guilt aversion. Despite the additional complexities such preferences introduce, many of the results from the material preference game carry over to that with guilt. Some new and intuitive results also emerge.

Guilt may help coordination as it facilitates the existence of completely mixed symmetric equilibria (CMSE), which are easier to coordinate on than many other types of equilibria. Guilt increases (decreases) participation when players are coordinated on high (low) participation equilibria. In a volunteer’s dilemma with guilt, group size has a non-monotonic effect on participation in games with high participation costs. Re-analysing existing experimental data, our equilibrium characterisation suggests that a low but positive guilt sensitivity parameter can explain observed participation rates.

To appreciate the relevance of our results it is important to note critical assumptions that they are predicated on. One such assumption is the solution concept. Given our focus on understanding coordination, our theoretical analysis was restricted to CMSE. While we have fully characterised the effects of guilt for this class of equilibria it is not obvious that the intuitions driving our results would extend to other classes of equilibria. For instance, would guilt aversion imply existence of asymmetric mixed strategy equilibria when none exist with material preferences? We leave it for future work to characterise the effect of guilt on the full set of equilibria.

Since our motivation was to study the effects of guilt on participation, we did not consider other motivations. However, by understanding the incentives behind our results

⁶Ederer & Stremitzer (2017) find θ estimates in the range of 0 to 20. However their study is less comparable as they allow utilities to be concave in guilt and their model allows promises to be made.

on guilt one can deduce the effect of other preferences in participation games. For instance, the reason for non-existence of CMSE in the material game was insufficient incentives to participate. Guilt aversion overcame this, but presumably so could a model with altruistic agents, for example. It would be interesting to more fully compare and contrast the differences between different preference models and test between them experimentally.

To the best of our knowledge, no other paper has empirically tried to identify the effect of guilt in participation games. Our empirical estimates provide a useful first-pass check of the portability of guilt aversion and its empirical relevance for participation games; however, they are far from conclusive. For example, it may be that rather than playing CMSE subjects are playing asymmetric equilibria and alternating between them such that they generate the same overall participation rate as a CMSE. Future empirical work should study individual subject choices rather than aggregate participation rates as we do.

Our empirical exercise relied on analysing existing experimental data. This limited our ability to test some of the novel hypotheses that emerged from our theory. For instance, our model suggests that in a volunteer's dilemma with guilt, group size can have a non-monotonic effect on participation. However, since we are not aware of any existing experimental work that has implemented a high cost volunteer's dilemma where group size is varied by treatment, our conjecture remains untested.

There is much to be understood on how guilt affects participation in the provision of public goods. We hope that the equilibrium characterisation presented here provides a useful starting point for further study, both theoretical and empirical.

Appendix: Proofs

Proof of Proposition 1

If player j chooses to participate, his expected material payoff is $(1 - F(w - 2; n - 1, p))v - c$. When $m < w$, j 's material payoff is $-c$. Let \underline{S} denote the set of strategy profiles such that the public good is not provided: $\underline{S} \equiv \{s \in S \mid m = \sum s_i < w\}$. Let $\underline{s} \in \underline{S}$. Then player j 's disappointment, if the public good provision threshold is not reached, is

$$\begin{aligned} D_j(\underline{s}, 1, \alpha_j) &= \max\{0, (1 - F(w - 2; n - 1, p))v - c - (-c)\} \\ &= (1 - F(w - 2; n - 1, p))v. \end{aligned}$$

If player j chooses to abstain, his expected material payoff is $(1 - F(w - 1; n - 1, p))v$. When $m < w$, j 's material payoff is 0 thus his disappointment is

$$\begin{aligned} D_j(\underline{s}, 0, \alpha_j) &= \max\{0, (1 - F(w - 1; n - 1, p))v - 0\} \\ &= (1 - F(w - 1; n - 1, p))v. \end{aligned}$$

Player i only feels guilty if he chose to abstain and is pivotal for provision, that is, when $m = w - 1$. Thus i 's expected utility from participating is $(1 - F(w - 2; n - 1, p))v - c$. Note that the probability of $m = w - 1$ is $\rho(w; n; p)$ and that if i abstains, then when $m = w - 1$ there are $n - w$ others abstaining and $w - 1$ others participating. This implies i 's expected utility from abstain is

$$(1 - F(w - 1; n - 1, p))v - \rho\theta v [(w - 1)(1 - F(w - 2; n - 1, p)) + (n - w)(1 - F(w - 1; n - 1, p))].$$

In a CMSE i 's expected utilities for participating and abstaining must be equal. Equating the two and simplifying gives

$$\rho(1 + \theta [(n - 1)(1 - F(w - 2; n - 1, p)) - (n - w)\rho]) = \frac{c}{v}. \quad (13)$$

Note that when $m = w - 1$ the difference in disappointment between those that participate and those that abstain is $\rho(w; n; p)$. Using this one can rewrite (13) as (11). ■

Proof of Proposition 2

Substituting $w = 1$ into (11) and using $q = 1 - p$ gives,

$$(1 + \theta(n - 1)(1 - q^{n-1})) q^{n-1} = \frac{c}{v}. \quad (14)$$

We first find an explicit expression for $q^*(c, n, v, \theta)$ then demonstrate its image lies in the unit interval. Implicitly differentiating (14) with respect to c gives

$$\frac{\partial q^*(c, n, v, \theta)}{\partial c} = \frac{1}{v(n - 1)q^*(c, n, v, \theta)^{n-2} [1 + \theta(n - 1)(1 - 2q^*(c, n, v, \theta)^{n-1})]}. \quad (15)$$

Solving partial differential equation (15) gives the general solution

$$q^*(c, n, v, \theta) = \left(\frac{2(K(n, v, \theta) + c)}{v(1 + \theta(n - 1)) + \sqrt{v(v + \theta(n - 1)) [v(n - 1)\theta - 4(K(n, v, \theta) + c - \frac{v}{2})]}} \right)^{\frac{1}{n-1}} \quad (16)$$

where $K(n, v, c)$ is a function. Substituting (16) into (14) and solving for $K(n, v, \theta)$, establishes that $K(n, v, \theta) = 0$. The particular solution of (15) is thus

$$q^*(c, n, v, \theta) = \left(\frac{2c}{v(1 + \theta(n - 1)) + \sqrt{v(v + \theta(n - 1)) [v(n - 1)\theta - 4(c - \frac{v}{2})]}} \right)^{\frac{1}{n-1}}. \quad (17)$$

We now demonstrate that $q^* \in (0, 1)$. Suppose $q^* \leq 0$. This would imply that

$$v(n-1)^2\theta^2 + (1-4c+2v)(n-1)\theta + v+1 \leq 0. \quad (18)$$

The LHS of (18) is a quadratic in θ . If the discriminant were negative then (18) would be false. So suppose that it is non-negative; the larger root of the quadratic is then

$$\frac{4c-2v-1 + \sqrt{16c^2 - (16v+8)c + 1}}{2v(n-1)}.$$

For this root to be non-negative it must be that $4v(1+v) \leq 0$, which is false. Thus $q^* > 0$. Now suppose $q^* > 1$. This would imply that $4c(c-v) > 0$, which is false since $c < v$ by assumption, thus $q^* < 1$. ■

Proof of Proposition 3

We first establish some properties of the LHS of equilibrium condition (11), then state how these can be used to identify when equilibria exist.

Condition (11) can be written as

$$\begin{aligned} \binom{n-1}{w-1} p^{w-1} (1-p)^{n-w} (1 + \theta[(n-1)(1 - \sum_{i=0}^{w-1} \binom{n-1}{i} p^i (1-p)^{n-i}) \\ + (w-1) \binom{n-1}{w-1} p^{w-1} (1-p)^{n-w}]) = \frac{c}{v}. \end{aligned} \quad (19)$$

Let $g(p, \theta)$ denote the LHS of (19). Clearly $g(0, \theta) = g(1, \theta) = 0$. We can show that

$$\begin{aligned} \frac{\partial g(p, \theta)}{\partial p} = \binom{n-1}{w-1} p^{w-2} (1-p)^{n-w-1} [\alpha + \theta[(n-w)\alpha + (n-1) \sum_{i=0}^{w-1} \binom{n-1}{i} p^i (1-p)^{n-i} (np-i-\alpha) \\ + 2(w-1) \binom{n-1}{w-1} p^{w-1} (1-p)^{n-w} \alpha]], \end{aligned}$$

where $\alpha = w-1-(n-1)p$. Note that when p is arbitrarily close to zero, then $\partial g(p, \theta)/\partial p > 0$ and that when p is arbitrarily close to one, then $\partial g(p, \theta)/\partial p < 0$ (although $(np-i-\alpha) < 0$, it is multiplied by an arbitrarily small number).

Next we argue that $g(p, \theta)$ is strictly quasiconcave in p . Rewrite equilibrium condition (11) as

$$\rho(1 + \theta[(n-w)(1 - F(w-1; n-1, p^*)) + (w-1)(1 - F(w-2; n-1, p^*))]) = \frac{c}{v}.$$

Note that the LHS equals $g(p, \theta)$. In order to show that $g(p, \theta)$ is strictly quasiconcave in p , reason as follows. Clearly ρ is a strictly quasiconcave function of p on $[0, 1]$. Note

that $H(p) = (1 + \theta[(n - w)(1 - F(w - 1; n - 1, p^*)) + (w - 1)(1 - F(w - 2; n - 1, p^*))]$ is a monotone increasing function of p , since $1 - F$ is increasing in p . Note also that $H(p)$ is bounded above by $1 + \theta(n - 1)$ and below by 1. This is sufficient for $g(p, \theta)$ to be strictly quasi-concave in p for $p < \arg \max \rho = \frac{w-1}{n-1}$. In order to guarantee $g(p, \theta)$ is strictly quasiconcave in p for $p > \arg \max \rho = \frac{w-1}{n-1}$, we demonstrate that the inflection point of $H(p)$ equals the argmax of ρ . The second derivative of $H(p)$ at $\frac{w-1}{n-1}$ is

$$H''\left(\frac{w-1}{n-1}\right) = \left(-\frac{1}{w(w^2-1)}\right)(n-1)^{3-n} \left((w-1)(n-1) \sum_{k=0}^{\infty} \frac{(2)_k(w+1-n)_k(1-w)^k}{(w+2)_k(n-w)^k k!}\right. \\ \left.+ \theta(n-w)^{n-2-w}(w-1)^w \sum_{k=0}^{\infty} \frac{(1)_k(w-n)_k(1-w)^k}{(w+1)_k(n-w)^k k!}\right) \left[(n-w) \binom{n-1}{w-1} - w \binom{n-1}{w}\right]$$

where $(c)_0 = 1$ and $(c)_k = \prod_{i=0}^{k-1} (c+i)$. Note that we can rewrite $[\cdot]$ as follows

$$(n-w) \binom{n-1}{w-1} - w \binom{n-1}{w} = \frac{(n-w)(n-1)!}{(w-1)!(n-w)!} - \frac{w(n-1)!}{w!(n-w-1)!} \\ = (n-1)! \left(\frac{1}{(w-1)!(n-w-1)!} - \frac{1}{(w-1)!(n-w-1)!} \right) = 0.$$

Thus $H''(\frac{w-1}{n-1}) = 0$ and the inflection point of $H(p)$ equals the argmax of ρ . Hence $g(p, \theta)$ is strictly quasiconcave in p and thus intersects $\frac{c}{v}$, twice, once or zero times.

Finally, note that since $[\cdot]$ in (19) is strictly positive, $\partial g(p, \theta)/\partial \theta > 0$. Thus for θ sufficiently high, $g(p, \theta)$ will intersect $\frac{c}{v}$ twice.

(a) If $\frac{c}{v} \geq \rho(w; n, \frac{w-1}{n-1})$, at least one CMSE exists when $\theta = 0$ (Observation 3). Given the properties of g established in this proof, there will exist two CMSE for all $\theta > 0$.

(b) If $\frac{c}{v} < \rho(w; n, \frac{w-1}{n-1})$, then no CMSE exists when $\theta = 0$ (Observation 3). By the properties of g established in this proof, there exist two CMSE if and only if θ is sufficiently high. ■

Proof of Proposition 4

Consider how $q^*(c, n, v, \theta)$ varies with each of its arguments in turn.

(a) First consider $\partial q^*/\partial c$. Clearly $\partial q^*/\partial c \neq 0$ given (15). Suppose then that $\partial q^*/\partial c < 0$. This requires that $[\cdot]$ in the denominator of (15) is strictly negative. Substituting (17) into $[\cdot]$, $[\cdot] < 0$ implies

$$\frac{1}{2} + \frac{1}{2\theta(n-1)} < \frac{2c}{v(1 + \theta(n-1)) + v(v + \theta(n-1)[v(n-1)\theta - 4(c - v/2)])}. \quad (20)$$

The LHS of (20) is positive, thus if the denominator of the RHS of (20) were negative, (20) would not be true. The denominator of the RHS of (20) must thus be positive. Simplifying (20) given this implies

$$v < c \left[\frac{4(n-1)\theta}{(\theta(n-1)+1)^2} \right]. \quad (21)$$

Note that $[4(n-1)\theta/(\theta(n-1)+1)^2] < 1$ if $(n-1)^2\theta^2 - 2(n-1)\theta + 1 > 0$. Since the discriminant of this quadratic in θ equals 0 and $(n-1)^2 > 0$, $(n-1)^2\theta^2 - 2(n-1)\theta + 1 > 0$ holds for all c, n, v and θ . However, this implies that (21) cannot hold as $[\cdot] < 1$ and $v > c$ by assumption. Thus our supposition is false and it must be that $\partial q^*/\partial c > 0$.

Also consider $\partial q^*(c, n, v, \theta)/\partial v$. Implicitly differentiating (14) with respect to v ,

$$\frac{\partial q^*(c, n, v, \theta)}{\partial v} = \frac{q^*(c, n, v, \theta)^{n-1}[1 + \theta(n-1)(1 - q^*(c, n, v, \theta))^{n-1}]}{-v(n-1)q^*(c, n, v, \theta)^{n-2}[1 + \theta(n-1)(1 - 2q^*(c, n, v, \theta)^{n-1} - 1)]}. \quad (22)$$

The numerator of (22) must be positive given (17). Since $\partial q^*/\partial v$ clearly cannot equal zero, suppose $\partial q^*/\partial v > 0$. This requires that $[\cdot]$ in the denominator of (22) be negative. Substituting (17) into $[\cdot]$ implies (20). However we have already established that (20) is false. Thus it must be that $\partial q^*/\partial v < 0$.

(b) Second consider how $q^*(c, n, v, \theta)$ varies with n . Assume that n is continuous to simplify the analysis. Implicitly differentiating (14) with respect to n gives

$$\begin{aligned} \frac{\partial q^*(c, v, n, \theta)}{\partial n} &= -\frac{q^*(c, v, n, \theta)}{n-1} \left[\frac{\theta(1 + q^*(c, n, v, \theta)^{n-1})}{1 + \theta(n-1)(1 - 2q^*(c, n, v, \theta)^{n-1})} + \ln q^*(c, v, n, \theta) \right]. \end{aligned} \quad (23)$$

The sign of (23) clearly depends on the sign of $[\cdot]$. Sign each term in $[\cdot]$ as follows. The numerator of the first term is strictly positive. Note that the denominator of the first term is identical to $[\cdot]$ in (15), and we have demonstrated that this is strictly positive (see discussion around (20)). Thus the first term in $[\cdot]$ of (23) is strictly positive. The second term is strictly negative. Thus the sign of (23) depends on which term is larger. Note that since $\lim_{c \rightarrow 0} q^* = 0$, it must be that $\lim_{c \rightarrow 0} [\cdot] = -\infty$. Thus there exists \bar{c} , such that if $c \leq \bar{c}$, then $\partial q^*/\partial n > 0$.

(c) Finally, consider $\partial q^*(c, n, v, \theta)/\partial \theta$. Implicitly differentiating (14) with respect to θ gives

$$\frac{\partial q^*(c, n, v, \theta)}{\partial \theta} = \frac{q^*(c, n, v, \theta)(q^*(c, n, v, \theta)^{n-1} - 1)}{1 + \theta(n-1)(1 - 2q^*(c, n, v, \theta)^{n-1})}. \quad (24)$$

Clearly this cannot equal zero. Suppose that $\partial q^*/\partial \theta > 0$. Since the numerator of (24) is negative, it must then be that the denominator is also negative. Substituting (17) into the denominator of (24), this requires that (20) is true. However, we have already established that (20) is false. Therefore $\partial q^*/\partial \theta < 0$. ■

Proof of Proposition 5

Let $g(n, w, \theta, p)$ denote the LHS of equilibrium condition (11). Recall that in the proof of proposition 3 we demonstrated that $g(n, w, \theta, 0) = g(n, w, \theta, 1) = 0$, that g is strictly quasi-concave and that $\arg \max_p g \in (0, 1)$.

(a) Equilibrium requires $g(n, w, \theta, p) = \frac{c}{v}$. Note that the LHS is independent of $\frac{c}{v}$. Given the properties of g stated at the start of this proof, an increase in $\frac{c}{v}$ increases p_L^* and decreases p_H^* .

(b) Let $p^*(n)$ denote an equilibrium probability as a function of n (with $p_L^*(n)$ and $p_H^*(n)$ defined analogously). We will demonstrate that $p_L^*(n+1) < p_L^*(n)$.

Given that we are interested in cases where there are two equilibria and given the properties of g stated at the start of this proof, it must be that $p_L^*(n) < \arg \max_p g(n+1, w, \theta, p)$. For all $p < \arg \max_p g(n+1, w, \theta, p)$, consider $g(n+1, w, \theta, p) - g(n, w, \theta, p)$.

$$\begin{aligned} & g(n+1, w, \theta, p) - g(n, w, \theta, p) \\ &= \rho(w; n+1, p) - \rho(w; n, p) \\ & \quad + \theta n [\rho(w; n+1, p)(1 - F(w-1, n, p)) - \rho(w; n, p)(1 - F(w-1, n-1, p))] \\ & \quad + \theta \rho(w; n, p)(1 - F(w-1, n-1, p)) + \theta(w-1) [\rho(w; n+1, p)^2 - \rho(w; n, p)^2] > 0. \end{aligned}$$

To understand why the expression is strictly positive consider each line in turn. The first line on the RHS is the first-difference of the pmf of a binomial distribution, which is strictly increasing in n for the interval of p of interest; thus the first line is strictly positive. Using the same property of the pmf, note that $\rho(w; n+1, p) > \rho(w; n, p)$ in the second line on the RHS. Also, since the cdf of the binomial distribution is decreasing in n , $1 - F(w-1, n, p) > 1 - F(w-1, n-1, p)$. Taken together, this implies that the second line of the RHS is strictly positive. The first term on the third line of the RHS is the product of three strictly positive numbers, thus is strictly positive itself. Finally, the aforementioned property of the pmf implies that the final term is strictly positive.

Equilibrium condition (11) requires that $g(n+1, w, \theta, p_L^*(n+1)) = g(n, w, \theta, p_L^*(n))$. Given that $g(n+1, w, \theta, p) - g(n, w, \theta, p) > 0$, it must be that $p_L^*(n+1) < p_L^*(n)$ since g is strictly increasing in p for the range of interest. Applying the above reasoning recursively establishes the result for arbitrary increases in n .

Analogous reasoning shows that $p_H^*(n+1) - p_H^*(n) < 0$.

(c) Let $p^*(w)$ denote an equilibrium probability as a function of w (with $p_L^*(w)$ and $p_H^*(w)$ defined analogously). We will demonstrate that $p_L^*(w+1) > p_L^*(w)$.

Given that we are interested in cases where there are two equilibria and given the properties of g stated at the start of this proof, it must be that $p_L^*(w) < \arg \max_p g(n, w, \theta, p)$.

For all $p < \arg \max_p g(n, w, \theta, p)$, consider $g(n, w + 1, \theta, p) - g(n, w, \theta, p)$.

$$\begin{aligned}
& g(n, w + 1, \theta, p) - g(n, w, \theta, p) \\
&= \rho(w + 1; n, p) - \rho(w; n, p) \\
&\quad + \theta(n - 1)[\rho(w + 1; n, p)(1 - F(w, n - 1, p)) - \rho(w; n, p)(1 - F(w - 1, n - 1, p))] \\
&\quad + \theta w[\rho(w + 1; n, p)^2 - \rho(w; n, p)^2] + \theta \rho(w; n, p)^2, \\
&= f(w; n - 1, p) - f(w - 1; n - 1, p) \\
&\quad + \theta(n - 1) \left[f(w; n - 1, p) \left(\sum_{k=w+1}^{n-1} f(k; n - 1, p) \right) - f(w - 1; n - 1, p) \left(\sum_{k=w}^{n-1} f(k; n - 1, p) \right) \right] \\
&\quad + \theta w[f(w; n - 1, p)^2 - f(w - 1; n - 1, p)^2] \\
&\quad + \theta f(w - 1; n - 1, p)^2, \\
&= f(w; n - 1, p) - f(w - 1; n - 1, p) \\
&\quad + \theta(n - 1) \left[f(w; n - 1, p) \left(\sum_{k=w+1}^{n-1} f(k; n - 1, p) \right) \right. \\
&\quad \left. - f(w - 1; n - 1, p) \left(f(w; n - 1, p) + \sum_{k=w+1}^{n-1} f(k; n - 1, p) \right) \right] \\
&\quad + \theta w[f(w; n - 1, p)^2 - f(w - 1; n - 1, p)^2] \\
&\quad + \theta f(w - 1; n - 1, p)^2, \\
&= f(w; n - 1, p) - f(w - 1; n - 1, p) \\
&\quad + \theta(n - 1) (f(w; n - 1, p) - f(w - 1; n - 1, p)) \left(\sum_{k=w+1}^{n-1} f(k; n - 1, p) \right) \\
&\quad + \theta w f(w; n - 1, p)^2 - \theta(n - 1) f(w - 1; n - 1, p) f(w; n - 1, p) \\
&\quad + \theta f(w - 1; n - 1, p)^2 - \theta w f(w - 1; n - 1, p)^2 < 0.
\end{aligned}$$

To understand why the expression is strictly negative consider each line in turn. The fourth and third from last lines are strictly negative as the pmf of the binomial is strictly decreasing in w for relevant p . Using the same property and that $n - 1 \geq w$ implies the penultimate line is strictly negative. The final line is strictly negative as $w > 1$.

Equilibrium condition (11) requires that $g(n, w + 1, \theta, p_L^*(w + 1)) = g(n, w, \theta, p_L^*(w))$. Given that $g(n, w + 1, \theta, p) - g(n, w, \theta, p) < 0$, it must be that $p_L^*(w + 1) > p_L^*(w)$ since g is strictly increasing in p for the range of interest. Applying the above reasoning recursively establishes the result for arbitrary increases in w .

Analogous reasoning shows that $p_H^*(w + 1) - p_H^*(w) > 0$.

(d) Given that $\partial g / \partial \theta > 0$ and the properties of g stated at the start of this proof it must be that p_L^* is decreasing in θ and p_H^* is increasing in θ . ■

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